Effective Field Theory — Serie 7

Chiral Perturbation Theory — 28.5.2024

1. If we consider QCD with *four* massless flavors, how many Goldstone bosons would arise in the breaking of chiral symmetry?

2. The effective Lagrangian for the Goldstone bosons associated with spontaneous chiral symmetry breaking takes the form

$$\mathcal{L} = \frac{F^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U\rangle + \mathcal{O}(p^4)\,, \tag{1}$$

where $\langle \dots \rangle$ denotes the trace and the field $U(x)$ can be parameterized in terms of the Goldstone field π^a as

$$U(x) = \exp\left(\frac{2i}{F}\pi^a t^a\right)\,.$$

a.) Expand the Lagrangian to fourth order in the Goldstone fields π^a. For simplicity, work with two massless flavors, where $t^a = \sigma^a/2$.
Hint:

$$\exp(i\vec{\sigma}\cdot\vec{v}) = \cos(|\vec{v}|) + i\vec{\sigma}\cdot\vec{v}\,\sin(|\vec{v}|)/|\vec{v}| \tag{2}$$

b.) Derive the Feynman rules for the Goldstone propagator and for the interaction of four Goldstone bosons.

c.) Derive the Goldstone-boson scattering cross section, i.e. the cross section for the process

$$\pi^a(p_1)\,\pi^b(p_2) \to \pi^c(p_3)\,\pi^d(p_4)\,.$$

and show that it vanishes in the limit $s = (p_1+p_2)^2 \to 0$.

d.) *Bonus exercise:* Physical quantities should be independent of the parameterization of the matrix $U(x)$. Modify the above parameterization as

$$U(x) = \exp\left(\frac{2i}{F}\pi^a t^a\, g(\vec{\pi}^2/F^2)\right)\,,$$

with an arbitrary real function $g(x) = 1+\alpha\,x+\dots$ analytic at $x=0$. Show that the new parameterization yields the same scattering cross section.

3. To construct the effective Lagrangian, we added external sources for left-handed and right-handed vector currents to the QCD Lagrangian and transformed these as

$$r_\mu \to V_R\, r_\mu\, V_R^\dagger - i(\partial_\mu V_R)\, V_R^\dagger$$
$$l_\mu \to V_L\, l_\mu\, V_L^\dagger - i(\partial_\mu V_L)\, V_L^\dagger$$

a.) Show that with these transformations of the external fields the QCD Lagrangian becomes invariant under *local* chiral transformations $V_L(x)$ and $V_R(x)$.

b.) Show that the leading-power chiral perturbation theory Lagrangian (1) is invariant under local transformations if we replace the regular derivatives by covariant one

$$i\partial_\mu U \to iD_\mu U = i\partial_\mu U + r_\mu U - U\, l_\mu \,.$$